where ψ denotes E_x. Since the integral equation (12) is obtained by discarding those terms smaller by a factor of order $\alpha^2\Delta$, it is reasonable to expect that the relative errors in both the eigenvalue β and the eigenfunction ψ are of the same order. (Based on a perturbation theory, a proof of such relations has been given for the standard eigenvalue problem in the matrix form [14].)

In the case where $\langle E_x \rangle$ is much greater than $\langle E_y \rangle$, such as in a mode excited by an x-polarized wave, it is found from (6) that, in general, the ratio between the magnitudes of the three Cartesian components is

$$E_x : E_z : E_y \simeq 1 : O(\alpha\sqrt{\Delta}) : O(\alpha^2\Delta). \tag{13a}$$

From the relation $\nabla \times \boldsymbol{E} = -j\omega\mu_0 \boldsymbol{H}$, one can find that

$$H_y = \frac{\beta}{\omega\mu_0} E_x \{1 + O[\alpha^2\Delta]\}. \tag{13b}$$

Then, H_y satisfies (12), under the same order of inaccuracy. The arguments made above can be given for E_y by simply interchanging the subscripts x and y. In summary, (12) is valid for the transverse Cartesian components, E_x, E_y, H_x, and H_y, whereas it does not hold for the axial components E_z and H_z. On applying the operator $(\nabla_t^2 - \gamma^2)$ to both sides of (12), we obtain (3), the differential equation in the scalar form.

IV. Conclusions

From the electric field integral equation a quantitative analysis of the effect of polarization charge has been given. It is found that the error due to the scalar approximation (neglecting the polarization charge) is proportional to the difference Δ, regardless of the functional behavior of the profile P. Physically, this fact is accounted for by noting that, so long as the difference Δ is kept small, a rapidly varying permittivity distribution leads to closely clustered polarization charge (positive or negative); hence the polarization effect is weakened due to self-cancellation. We have conducted many calculations for a circular fiber (using the method in [15]), and all the results support the conclusion.

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Saturation of the SIS Mixer by Out-of-Band Signals

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Abstract —**The tendency of SIS mixers to saturate at low input signal levels is shown to depend on the total signal voltage across the junction, including frequency components outside the band of interest. If large dynamic range is to be achieved, mixers should be designed with embedding networks that present low impedances to the junction at out-of-band frequencies.**

I. Introduction

SIS (superconductor–insulator–superconductor tunnel junction) mixers allow the construction of very sensitive receivers at millimeter wavelengths, but the dynamic range of such receivers may be limited because of mixer saturation at low input powers. This has long been recognized as a significant problem [1]–[5], and approximate formulas have been presented for the input power at which departure from linear operation begins [1], [2]. Reports of experimental mixers often include measurements of this saturation power (e.g. [4], [5]). However, nearly all of this theoretical and experimental work has considered only a monochromatic input signal. In practice, it is often necessary for the receiver to accept a broad-band noise signal, such as thermal noise at room temperature. For example, strong noise sources are often used to calibrate the gain of the receiver and to determine its noise temperature; unless it can be assured that the receiver remains linear for these signals, the calibration will be in error. We will show here that it is inaccurate to assume that the saturation noise temperature T_{sat} for broad-band signals will be such that $P_{sat} = kT_{sat}B$, where P_{sat} is the saturation power measured for monochromatic signals and B is the receiver's bandwidth. This is because the broad-band signal contains power well outside this bandwidth, and, unless special precautions are taken, an SIS mixer will begin saturating because of the out-of-band signals well before the in-band power reaches P_{sat}.

II. Approximate Argument

An argument explaining the saturation mechanism of SIS mixers was first put forward by Smith and Richards [1], and later developed into an explicit formula [2]. The idea is that the small-signal gain of the mixer is a function of its dc bias, and reaches local maxima at certain voltages (photon peaks) where the mixer is normally operated. If the output frequency (IF) is low, then the output signal voltage may be considered a perturbation of the bias voltage, so that the instantaneous gain varies over the IF cycle. As the signal voltage gets large, the average gain is reduced from the peak. The embedding impedances required for low-noise, high-gain operation of an SIS mixer are such that the largest signal voltage is likely to occur at the IF, in which case this argument gives a fair description of the saturation mecha-

Manuscript received September 15, 1987; revised December 31, 1987.

The author is with the National Radio Astronomy Observatory, 2015 Ivy Road, Charlottesville, VA 22903. The Observatory is operated by Associated Universities, Inc., under contract with the National Science Foundation

IEEE Log Number 8820445.

nism. The onset of saturation can be estimated quantitatively if the gain-versus-bias function is known; this can be found from the small-signal theory, which can then be used to relate the IF signal voltage to the input signal power [2].

More generally, the onset of saturation occurs when the total signal voltage (at all frequencies) appearing across the SIS junction can no longer be treated as a small perturbation of the dc bias and local oscillator voltages. A quantitative treatment of this is extremely difficult, even with some simplifying assumptions. However, it is easy to see that the voltage due to out-of-band signals can exceed that due to in-band signals in practical cases. Suppose that the mixer's RF bandwidth (frequency range over which the RF source impedance seen by the junction is nearly constant) exceeds the IF bandwidth (constant load impedance), and that the RF source has constant noise temperature. Then the IF voltage spectrum will follow the IF load impedance, which can easily be an open circuit at some out-of-band frequencies. Typically, the junction is connected by a 50-Ω transmission line to an IF amplifier whose input impedance is nominally 50 Ω over its design bandwidth, but is mainly reactive outside this bandwidth. If the transmission line is at least a few cm long, then the phase of the reflection coefficient seen by the junction will vary by 2π over a few GHz. Since the RF bandwidth of SIS mixers typically exceeds a few GHz, the largest spectral voltages appearing across the junction are likely to be at those out-of-band IF's where the load impedance is nearly an open circuit.

This problem can be avoided by a circuit design that is not typical of SIS receivers so far built. If a bandpass filter covering the desired IF band is placed very close to the SIS junction (at a distance small compared to v/B_{RF}, where v is the local propagation velocity and B_{RF} is the RF bandwidth), and if the filter is designed to provide low out-of-band input impedance (shunt input resonator), then the out-of-band voltage can be kept small.

III. Analysis Based on Quantum Mixing Theory

The quantum mixing theory of Tucker [6] begins with an expression for the total current in an SIS junction induced by any applied time-varying voltage, and proceeds by letting the voltage be

$$V(t) = V_{\mathrm{dc}} + V_L \cos 2\pi f_L t + v_s(t) \tag{1}$$

where V_{dc} is the dc bias, V_L is the amplitude of the (large) local oscillator at frequency f_L, and $v_s(t)$ is the (small) signal of interest. The response can be analyzed for a monochromatic signal by letting

$$v_s(t) = \sum_{m=-\infty}^{\infty} v_m \cos(2\pi f_m t + \phi_m) \tag{2}$$

where $f_m = mf_L + f_o$ for output frequency f_o, and where one of the terms is due to the input signal source and the others are mixing products. If all of the amplitudes $\{v_m\}$ are sufficiently small, then several simplifications are possible. First, the total current induced by the signal voltages can be computed as the superposition of the current induced by each, because second-order mixing (cross products) can be neglected. Second, the amplitude of the current i_n at frequency f_n induced by the voltage term at frequency f_m can be expanded as a Taylor series in v_m with all but the first-degree term being negligible. And third, the addition of another small voltage at a frequency not in $\{f_m\}$ (an "out-of-band" frequency) will not affect the induced current at any frequency in $\{f_m\}$. These first two facts allow the pumped junction to be treated as a linear network with admittance matrix elements $Y_{nm} = i_n/v_m$, and the third fact allows out-of-band frequencies to be neglected.

It is possible to relax the small-signal assumption, but the complete analysis of the SIS mixer performance then becomes very difficult. Nevertheless, an analytical demonstration of the effects of large signals can be made using the equations of the quantum tunneling theory. We will compute only the current induced in the junction for given signal voltages, without regard for the external circuit. It will be seen that large out-of-band signal voltages can affect the in-band currents by two mechanisms, and that these mechanisms are in addition to the nonlinearity of in-band current versus in-band voltage; the latter is the only saturation mechanism applicable to the monochromatic case considered by earlier authors.

If we now use

$$v_s(t) = \sum_{m=-\infty}^{\infty} \left[v_m \cos(2\pi f_m t + \phi_m) + v'_m \cos(2\pi f'_m t + \phi'_m) \right] \tag{3}$$

in (1), where f_m is an in-band frequency and $f'_m = mf_L + f'_o$ is an out-of-band frequency, then it is shown in the Appendix that the junction current can be written

$$I(t) = \mathrm{Re} \sum_l \sum_{l'} J_l(\alpha_L) J_{l'}(\alpha_L) \prod_m \sum_{k_m} \sum_{k'_m} J_{k_m}(\alpha_m) J_{k'_m}(\alpha_m) \cdot \sum_{p_m} \sum_{p'_m} J_{p_m}(\alpha'_m) J_{p'_m}(\alpha'_m) e^{i2\pi(\delta l f_L + \delta k_m f_m + \delta p_m f'_m)t} \cdot e^{i(\delta k_m \phi_m + \delta p_m \phi'_m)} \tilde{I}(f_L l' + f_m k'_m + f'_m p'_m) \tag{4}$$

where $\alpha_L = qV_L/hf_L$, $\alpha_m = qv_m/hf_m$, $\alpha'_m = qv'_m/hf'_m$, q is the electronic charge, h is Planck's constant, J_k is the kth-order Bessel function of the first kind, and $\tilde{I}(f)$ is the analytic signal associated with the dc current–voltage characteristic of the junction (see the Appendix for a precise definition). The limits of the product and of all sums in (4) are $-\infty$ to ∞, and $\delta l = l - l'$, $\delta k_m = k_m - k'_m$, $\delta p_m = p_m - p'_m$. This result is derived from the general formula of quantum tunneling theory [6] without assuming that the amplitudes v_m and v'_m are small.

Although (4) appears to be quite complicated, its essential features are these: the time variation is contained in complex exponentials at each possible mixing frequency of the constituent ac voltages, and the amplitude of the current at each frequency is given by a sum of Bessel functions of the voltage amplitudes along with samples of the (analytic signal of) the dc $I-V$ curve.

Careful study of (4) leads to the following conclusions. The first is that first-order mixing terms, i.e., those resulting from any harmonic of the (large) LO signal and the fundamental frequency of one of the small-signal factors, have an amplitude whose lowest order term is proportional to $J_0(\alpha_m)J_1(\alpha_m) = \frac{1}{4}\alpha_m(1 - \frac{5}{24}\alpha_m^2 + O(\alpha_m^4))$. These terms include the desired mixing products. The second conclusion is that if all out-of-band voltages are zero, the factors involving them are unity; if not, then those factors reduce the amplitude of the first-order mixing terms by $J_0(\alpha'_m)^2 = 1 - \frac{1}{4}\alpha_m'^2 + O(\alpha_m'^4)$. This factor is of the same order as the nonlinear factor resulting from the in-band signals; thus, an out-of-band voltage causes the same amount of saturation of the in-band gain as would an in-band voltage of the same amplitude. If the out-of-band voltage is larger, perhaps because the embedding impedance is larger at that frequency, then it will be the dominant cause of saturation. The third conclusion is that for certain choices of the out-of-band IF f'_0, higher order mixing products involving the out-of-band signals can appear in-band. For example, if $f_o' = f_o/2$, then the second harmonic (a second-order product) has this property. For broad-band signals such as thermal noise, second-order intermodulation products of this type are incoherent with the desired signals and therefore appear as an increase in noise at high signal levels, rather than as a reduction in gain (saturation). However, third-order (and higher odd-order) mixing products can be coherent and can contribute to the saturation. This can be expected to be much less important than the effects noted in the first and second conclusions.

The use of (4) to analyze fully a given junction and embedding network is especially difficult. It is necessary to solve for the voltages and currents at all frequencies simultaneously, given the (linear) constraints imposed by the embedding network. The situation can be simplified by considering only the three-port model, where the embedding network presents a short circuit at all frequencies f_m, f_m' for which $|m|>1$; this leaves nonzero voltages at six frequencies, three in-band and three out-of-band. A further simplification would be to neglect all but first-order mixing products. A solution might then be obtained iteratively by first using the small-signal Y matrix to find the approximate signal voltages, then using these in (4) to estimate the currents, then using the currents in the embedding network to obtain improved approximations to the voltages, and repeating until convergence. This still would not treat the broad-band noise case. It remains a difficult calculation, and the author intends to pursue it in a future publication.

IV. Conclusions

It has been demonstrated by both analysis and intuitive argument that gain saturation in an SIS mixer results when the total signal voltage across the junction becomes too large. It is emphasized that this includes voltages at frequencies outside the bands of interest of the mixer, such as arise when the input is broad-band noise. To obtain the largest dynamic range, the designer must ensure that the embedding network suppresses such voltages. The network can do this by approaching a short circuit at out-of-band frequencies. In high-gain mixers, the largest voltages normally occur at the output frequency (IF); in such cases, a carefully designed IF filter can significantly improve the dynamic range.

Appendix

For an arbitrary time function of applied voltage $V(t) = V_{dc} + V_{ac}(t)$, the expected value of current in a tunnel junction is given by [7]

$$I(t) = 2\,\mathrm{Re}\left\{\int_{-\infty}^{t} I_{FT}(t-t')\, e^{i2\pi(q/h)\int_{t'}^{t} V_{ac}(\tau)\,d\tau}\, dt'\right\} \quad \text{(A1)}$$

where $I_{FT}(t)$ is the Fourier transform of the dc current–voltage characteristic of the junction $I_{dc}(V)$ with respect to transform variable $f = q(V - V_{dc})/h$:

$$I_{FT}(t) = \int_{-\infty}^{\infty} I_{dc}(V_{dc} + hf/q)\, e^{-i2\pi ft}\, df. \quad \text{(A2)}$$

This formula can be easily derived from [6, eqs. (2.8), (2.11), and (2.16)]. Then if $V(t)$ is given by using (3) in (1), (A1) becomes

$$I(t) = 2\,\mathrm{Re}\left\{\int_{-\infty}^{t} I_{FT}(t-t')\, F(\alpha_L, 0, f_L, t, t') \prod_{m=-\infty}^{\infty} F(\alpha_m, \phi_m, f_m, t, t')\, F(\alpha_m', \phi_m', f_m', t, t')\, dt'\right\} \quad \text{(A3)}$$

where

$$F(\alpha, \phi, f, t, t') = \sum_{k=-\infty}^{\infty} \sum_{k'=-\infty}^{\infty} J_k(\alpha) J_{k'}(\alpha) \cdot e^{i(k-k')(2\pi ft+\phi)} e^{i2\pi k' f(t-t')}. \quad \text{(A4)}$$

This result follows from carrying out the integral in the exponent of (A1) and using the identity

$$e^{ia\sin x} = \sum_{k=-\infty}^{\infty} J_k(a)\, e^{ikx}. \quad \text{(A5)}$$

Each term of the integrand of (A3) contains $I_{FT}(t-t')$ and an exponential factor involving $t - t'$, but all other factors are constant; carrying out this integral then leaves (4). The function $\tilde{I}(f)$, used in (4), is the analytic signal of $I_{dc}(V_{dc} + hf/q)$, given by

$$\tilde{I}(f) = 2\int_{0}^{\infty} I_{FT}(t)\, e^{+i2\pi ft}\, dt. \quad \text{(A6)}$$

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Variational Bound Analysis of a Discontinuity in Nonradiative Dielectric Waveguide

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Abstract —**This paper describes the application of the variational bound method to nonradiative dielectric waveguide for the analysis of scattering by a dielectric obstacle, in this case a rectangular, air-filled discontinuity in the dielectric center strip. Closed-form equations are obtained that can be used directly in the design of networks using reactive components, such as filters. Measured data agree well with the theoretical calculations.**

I. Introduction

The application of specific properties of discontinuities in waveguides forms the basis of a variety of microwave devices. In the nonradiative dielectric waveguide only one such analysis has been reported, by Yoneyama *et al.* [1], where a step discontinuity was described and applied in the design of a filter. Expressions for describing the network are not given.

In this paper, the variational bound (VB) method described by Aronson *et al.* [2] is used to analyze the scattering from a rectangular hole through the dielectric center conductor of the

Manuscript received March 3, 1987; revised January 23, 1988.

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IEEE Log Number 8821072.